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MEAN TEMPERATURE IN METEOROLOGY

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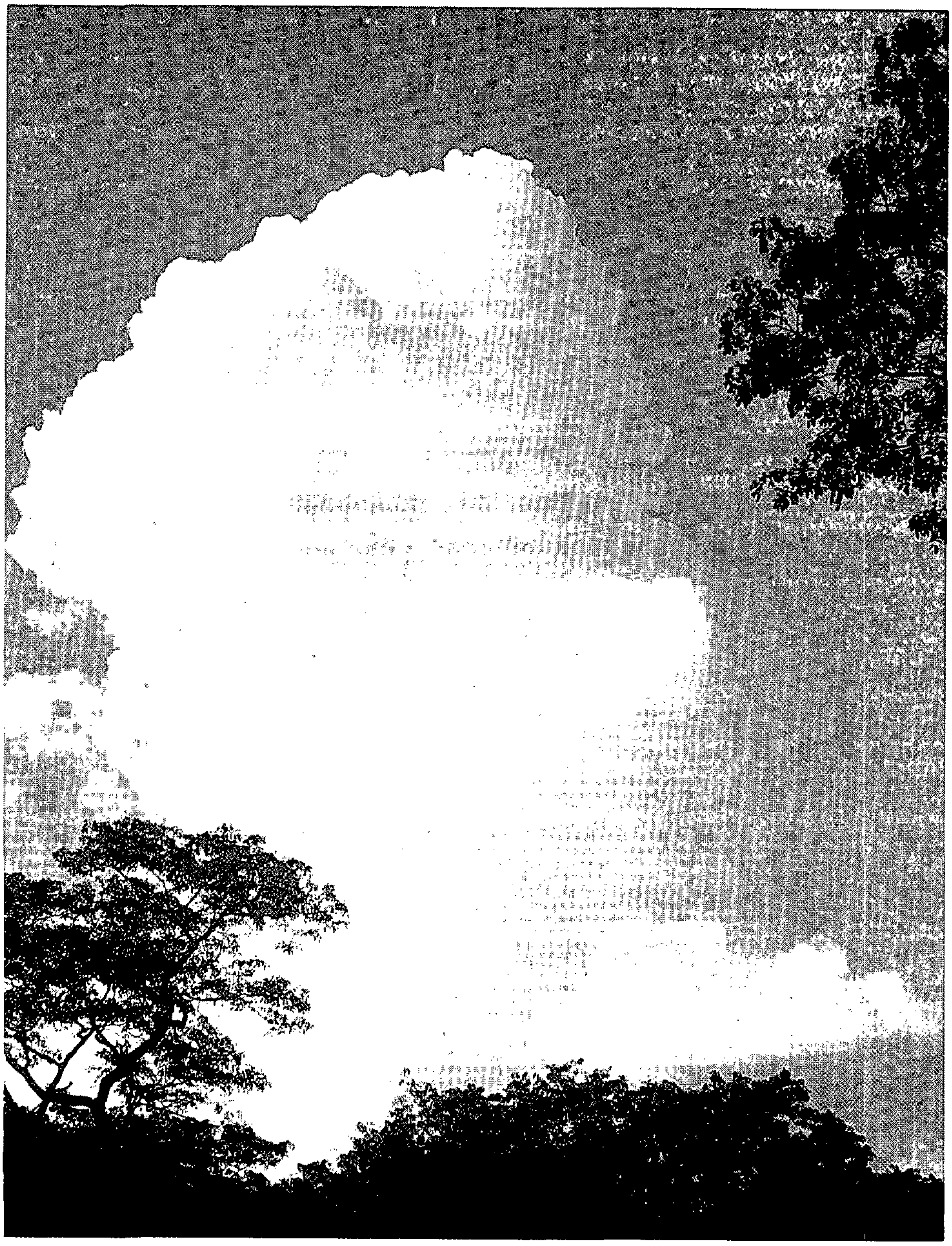

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MEAN TEMPERATURE IN METEOROLOGY

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OPPDRAGSGJEVAR

DNMI

SAMANDRAG

Written in 1975, this article illustrates the problems that confronted climatologists, long before the invention of any electronic equipment, when mean air temperature had to be found, and how these problems were tackled by, for example, the Norwegian Meteorological Institute (DNMI). This article also contains results of a study made by the present author concerning errors in the computed mean monthly air temperatures by using the usual practice of the DNMI. It is shown that those errors are quite trifling, and they are completely overshadowed by other error sources.

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1. Definition

If T is the temperature observed as a function of time τ, the strict definition of mean temperature T_m is

(1) $$T_m = \frac{1}{\tau_2 - \tau_1}\int_{\tau_1}^{\tau_2} T \, d\tau$$

2. How to find the mean temperature T_m

Method 1 T_m may be found from a thermogram by aid of a planimeter or other methods for measuring areas.

Method 2 If T has been observed by reading a thermometer with sufficiently small time intervals, all intervals being equal, T_m may be found as the arithmetical average of all the observed temperatures:

(2) $$T_m = \frac{1}{N}\sum_{i=1}^{i=N} T_i = \frac{1}{N}(T_1 + T_2 + \ldots T_N)$$

(Some climatologists have defined T_m according to (2) and fixed time intervals, usually 1 hour).

Method 1 requires an automatic record, small observational labour, but a rather laborious process before reliable T_m can be found.

Method 2 requires a thermometer, considerable observational labour, but rather less labour afterwards.

3. Labour-saving formulae

Climatologists are above all interested in mean temperatures for whole months. When it comes to such long times, quite acceptable values for T_m may be obtained from a whole network of meteorological stations by aid of very simple formulae, leading to an immense saving of labour. (Up to present time, at least, errors in T_m for a single month in a single year of 0.1 to 0.2°C have been considered quite acceptable). The simplest of these formulae is

$$T_{nx} = \frac{1}{2}(T_n + T_x)$$

where T_n is the average of the minimum temperatures for each day and T_x is the average of the maximum temperatures for each day. T_{nx} is then often used as if it were T_m. As a matter of fact, T_{nx} may differ from T_m with several tenths of a degree. The difference between them is especially sensitive to changes in environment, screening of the thermometers and the thermometer pattern. Working with T_{nx} for climatological purposes is therefore always tricky matters.

When thermometer observations are made at fixed hours, not necessarily with equal time intervals, one can readily find the averages appropriate of the actual hours. Let those averages for an actual month be T_1, T_2, T_3, ... One should expect that if the fixed hours are fairly equidistant in time, a satisfactory formula for T_m would be a linear expression:

$$T_{am} = a_1T_1 + a_2T_2 + a_3T_3 \ldots + c \tag{3}$$

where a_1, a_2, a_3, ... and c are constants appropriate for the month and the place.

Thus when Christopher J. Hansteen, founder of the astronomical observatory at Oslo, had started the long temperature series in 1837 from that place, he adopted a formula of this type. From March 1838 the fixed hours were 7, 9, 14, 16, and 22. It cannot be said that these fixed hours were equidistant in time, but by using suitable constants a, he obtained a result which he found acceptable.

The constants in formula (3) must be found by aid of the methods in section 2. in this article. Hansteen found his constants from hourly observations in 2-3 years 1841-1847. When DNMI started its work 1866, the fixed hours for its network were planned to be 8^h, 14^h and 20^h local mean time. Here was an equal time interval between 8^h-14^h and 14^h-20^h, but in the night the time interval was twice as long. As it was (nearly) impossible to establish 2^h in the night as a fixed hour, another way was found in order to improve the temperature statistics, and that was the introduction of the minimum thermometer in 1875.

If minimum temperatures shall be used in computing Tm, as a supplement to temperatures from the fixed hours, formula (3) must get a term kTn in addition:

$$T_{kam} = a_1T_1 + a_2T_2 + a_3T_3 \ldots + kT_n + c \tag{4}$$

The constants in the formula represent weight attributed to the actual observation. If we let the observations from the fixed hours carry equal weight and use 3 fixed hours, we can write:

$$a_1 = a_2 = a_3 = a$$

Introducing the average of T_1, T_2, and T_3: $M=\frac{1}{3}(T_1 + T_2 + T_3)$, neglecting c, and remembering that the weight sum 3a + k must be 1, we get

$$T_{kam3} = 3aM + kT_n = (1-k)M + kT_n = M - k(M-T_n)$$

or if we will use less cumbersome indices

$$T_{km} = M-k(M-T_n) \tag{5}$$

This formula, attributed to KÖPPEN (1846-1940), has been used by DNMI since 1891, and recalculating has been done for the years 1875-1890 with this formula. The constant k had to be found for every month and every place in the network. By aid of the methods in section 2. in this article Tm was found for a few selected stations in the network. A geographical pattern was discovered which rendered it possible to draw isopleths and find k for the other stations by interpolation.

It proved to be far from easy to stick to the original fixed observation hours. The development of communications from 1866 onward rendered local time more and more unpractical, until in 1895 Mean European Time (M.E.T) was ordered by law. The confusion in the observation hours was quite formidable in those years. A change in observation hours requires a revision of the constant k. When in 1920 the evening observation was fixed 1 hour earlier, so that the fixed observation hours thereafter should be 8^h, 14^h and 19^h M.E.T., a completely new table of k had to be worked out. Then in 1949 the fixed hours became 7^h, 13^h and 19^h, and another table had to be worked out. To complicate matters further, change has been made in the time when the minimum thermometer was read. This caused another change in k.

4. Magnitude and distribution of errors caused by the use of formula (5) ("k-formula") in computing mean temperature

If we had a perfect thermograph (which we of course have not), we could find the errors caused by the use of the k-formula (5) simply by comparing the results of that formula with the results obtained by using method 1. in section 2. in this article. In his great work "Mittel und Extreme der Lufttemperatur" p. 15-16 B. J. Birkeland writes about this problem and refers to investigations made by some authors. He gives tables with results of investigations which he himself must have taken part in. He has found errors up to 0.46°, and in fact much larger errors in his material from Oslo 1894-1931 than H. Mohn found in a material from 1884-1890.

After having seen this curious result, the present author made an investigation himself, which shall be outlined here.

We shall see that the poorer result of formula (5) which B. J. Birkeland found, is a complicated affair, and that the errors which he cites in the publication, are only partially due to this formula. There are also errors in the mean temperature found by using method 1. in section 2. in this article. Such errors may be due to one or more of the following causes:

1) The person working with the thermogram has made mistakes.
2) The thermograph has been mechanically unstable.
3) Faulty maintenance of the thermograph.
4) The thermograph has been moved from its proper place.

In the Yearbook of the DNMI 1896-1930 we find tables for Oslo containing temperature for each hour in one table, and temperature for each of the 3 fixed observation hours together with the minimum temperature in another table. The thermograms have been calibrated from day to day by aid of the thermometer readings at the fixed hours. Thus there is no point in comparing the temperatures in the two tables at the fixed hours, as these are made equal. But minimum temperatures in the two tables are independent, and these were studied by the present author 1974-1975. Instead of a pure random distribution of small differences, as had been expected, there were many grave systematic variations from time to time in addition. Thus in 1908 between April 22 and July 24 there was a strong anomaly in those differences. In June 1908 the average minimum temperature T_n is 10.5°, while the average temperature at 3^h is said to be 8.73°, and all the night hours from 1^h to 5^h have been colder than the minimum, a complete absurdity, hopelessly inconsistent. Obviously there is no point in using the mean temperature found by aid of thermograms as "correct" and the mean temperature found by using formula (5) as "erratic" in this case. Here was the error 0.46° which Birkeland cited. The Yearbook for 1923 contains a correction table for 1908 April 22 - July 24. This table cannot Birkeland have been aware of. Generally, there can be no point in using the mean temperature T_m found by aid of thermogram as check on the T_m found by using formula (5) when the difference between the minima read by thermometer and thermogram is anomalous.

At this stage I made the following plan:

1) All the T_m found by aid of thermogram, hereafter called T_{24M}, were arranged against the corresponding T_m found by aid of formula (5), hereafter called T_{kM}. The differences $T_{kM} - T_{24M}$ were tabulated and called ΔT_m.

2) By using the lowest temperature t_{gn} found in the table with hourly temperatures (these had been found by aid of thermogram) for each day between the hours when the minimum thermometer was read, the differences $t_{gn} - t_n$ were tabulated, t_n being the minimum temperature read on the minimum thermometer (presumably corrected for instrumental error). The monthly means ΔT_n of these differences were then computed. (Obviously ΔT_n should be positive, but this was far from being always so!).

3) For each month $\Delta T_m = T_{kM} - T_{24M}$ was plotted against ΔT_n on a diagram with ΔT_n as abscissa and ΔT_m as ordinate. One diagram was made for all the January months, another diagram for all the February months, and so on, 12 diagrams in all.

The result was that every diagram showed a well marked correlation between ΔT_m and ΔT_n, and graphs (regression lines) could be drawn (Fig. 1). All these sloped in the same direction, but somewhat more in the summer months than in the winter months. Negative ΔT_n (which should not have occurred!) tended to give positive ΔT_m and vice versa.

There is little doubt about the following conclusion:
The scattering about the regression lines represents the error distribution caused by the use of formula (5), not the

scattering about the abscissa axis or a parallel with that axis.

The next step was to correct all the differences ΔT_m, in order to separate the error sources. If a month is represented on a diagram by a point on a parallel with the ordinate axis having abscissa ΔT_n and the point having ordinate ΔT_m and the graph is crossing this parallel in a point with ordinate R, then ΔT_m - R is the corrected difference ΔT_m for this month, and shall be interpreted as the error in T_{kM} caused by use of formula (5) for this month. All these corrected differences were now tabulated, and the statistical analysis could be performed.

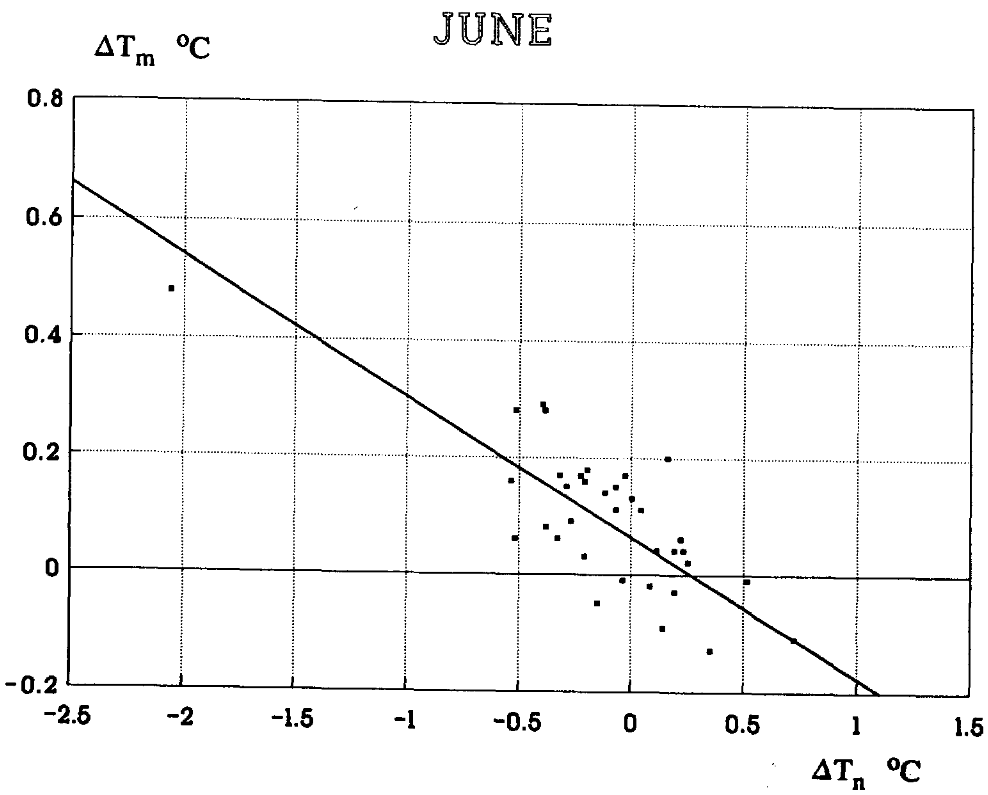

Fig. 1 One of the 12 diagrams showing correlation between ΔT_m and ΔT_n. The point representing June 1908 is near the upper left corner.

Contrary to H. Mohn and B. J. Birkeland I could find no clear yearly variation in the average magnitude of the corrected ΔT_m. I should think that H. Mohn found a variation because his material was too small and B. J. Birkeland because he had not corrected ΔT_m.

My result is that the average magnitude of the corrected ΔT_m was 0.04° to 0.07°, fluctuating irregularly through the year:

Jan	Feb	Mar	Apr	May	Jun	Year
0.048	0.057	0.058	0.043	0.049	0.069	
Jul	Aug	Sep	Oct	Nov	Dec	0.052
0.053	0.039	0.052	0.048	0.050	0.057	

Here 0.052° shall be interpreted as the real average error caused by the use of formula (5), the "k-formula".

It is interesting to see that this result is close to that found by H. Mohn: 0.055°.

All the 420 corrected ΔT_m were grouped for a frequency analysis. The distribution can be said to be strictly Gaussian or normal.

Only one of the 420 months had a ΔT_m of a magnitude larger than 0.2° (September 1923 with 0.23°), when the corrected ΔT_m were used.

Because the error distribution is Gaussian, the formulae appropriate of such distribution can be applied, and it is easy to see that errors caused by using formulae (5), the "k-formula", are much too small to be of any nuisance in climatological work.

5. References

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